

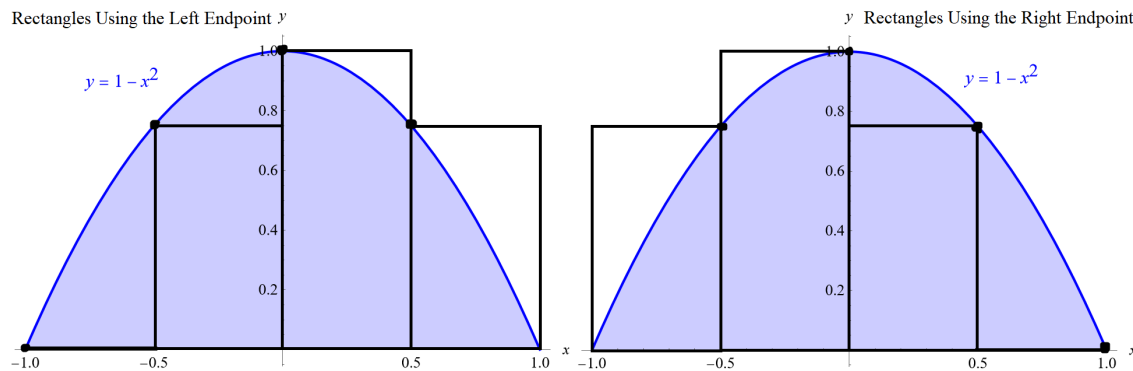
Exercise 29

For the following exercises, consider the function $f(x) = -x^2 + 1$.

Approximate the area of the region between the x -axis and the graph of f over the interval $[-1, 1]$.

Solution

The area below the graph of $f(x) = 1 - x^2$ can be found using rectangles in two ways. In the first way the top left of each rectangle touches the graph, and in the second way the top right of each rectangle touches the graph.



Add the areas of all the rectangles (each with width 0.5) in each case.

$$\begin{aligned}
 A_{\text{Left}} &\approx (0.5)f(-1) + (0.5)f(-0.5) + (0.5)f(0) + (0.5)f(0.5) \\
 &\approx (0.5)(1 - (-1)^2) + (0.5)(1 - (-0.5)^2) + (0.5)(1 - (0)^2) + (0.5)(1 - (0.5)^2) \\
 &\approx (0.5)(0) + (0.5)(0.75) + (0.5)(1) + (0.5)(0.75) \\
 &\approx 1.25
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{Right}} &\approx (0.5)f(-0.5) + (0.5)f(0) + (0.5)f(0.5) + (0.5)f(1) \\
 &\approx (0.5)(1 - (-0.5)^2) + (0.5)(1 - (0)^2) + (0.5)(1 - (0.5)^2) + (0.5)(1 - (1)^2) \\
 &\approx (0.5)(0.75) + (0.5)(1) + (0.5)(0.75) + (0.5)(0) \\
 &\approx 1.25
 \end{aligned}$$

The exact area is

$$A = \int_{-1}^1 (1 - x^2) dx = 2 \int_0^1 (1 - x^2) dx = 2 \left(1 - \frac{1}{3} \right) = \frac{4}{3} \approx 1.333.$$